An efficient numerical algorithm for solving the muscle recruitment problem in inverse dynamics simulations

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Introduction

Simulation of the musculo-skeletal system is a subject of importance to fundamental research as well as to practical engineering and medical applications. This contribution deals with inverse dynamics problems and a new algorithm for solving the muscle recruitment problem for this type of simulations has been developed. The method is implemented into a general body modeling and optimization software system called AnyBody (see for instance the model in Fig. 1).

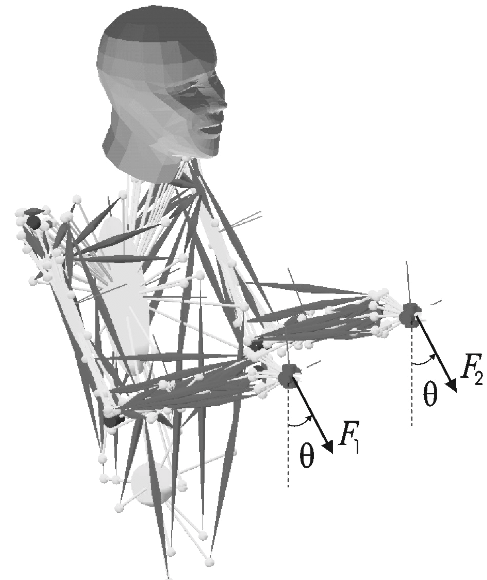

Figure 1: Model of the upper body in the AnyBody software system.

Detailed analysis typically involves estimation of the muscle and joint forces under given working situations. If the motion is prescribed completely, the problem can be solved by inverse dynamics methods and this is the case we shall focus on. The muscle recruitment problem is inherently haunted by the redundancy arising from the body having more actuators, i.e. muscles, than degrees of freedom. Mathematical optimization, applied to the distribution of muscle forces, is a well-known technique for simulating the muscle recruitment. Several criteria can be found in the literature, but minimization of a weighed sum of muscle loads raised to some power, the so-called polynomial criteria, have become popular. The polynomial criteria for powers higher than one and the majority of other criteria leads to non-linear optimization problems that require sophisticated and time-consuming numerical methods. In contrast, criteria leading to a linear optimization problem are generally attractive in terms of efficiency and ease of implementation.

This contribution presents the implementation of such a linear criterion. It can be considered as a minimum fatigue criterion, formulated as minimization of the maximal relative muscle load (see An et. al, 1984). This problem can be cast into the numerically attractive linear form; however, it has been shown that a min/max criterion may leave some muscle forces undetermined, see e.g. Bean et. al., 1988. In the following, we show that this can happen in three distinct cases and can be avoided by careful management of the problem variables.

The motivation for usage of this criterion is twofold: It is physiologically reasonable and it is numerically efficient. Regarding the former, it is clear that the linear, polynomial form, i.e., with the power equal to one, is not physiologically reasonable: For this case, the best suited muscles will carry the external load until they reach their limit, before other muscles are activated to assist. Higher powers provide that muscles collaborate in carrying the loads but it is difficult to establish experimentally which higher power is the better. The min/max criterion behaves much like high-power polynomial criteria and considered as a minimum fatigue criterion, it appears to be based a rational foundation, unlike the polynomial form.

Numerical efficiency has in this work been a motivation factor with several aims. Firstly and naturally, efficient computations are convenient when dealing with very large models, such as a full body model; during development, testing, and finally usage of a model, numerically efficiency allows the analysts to be efficient in their work. Secondly, numerical efficiency allows for systematic studies of input parameters or even optimization of the system with respect to these parameters. Parameter optimization, being one of our aims, can be used for several purposes such as design optimization of man-driven artifacts, model parameter estimation, and simulation of human motion patterns (see Rasmussen et. al., 2001a). These problems are all important issues and very computationally demanding.

In the following, we present the min/max muscle recruitment criterion and the new method for its solution. Special care is taken to resolve the indeterminacy in a proper manner and we shall demonstrate these features on numerical examples. The larger model comprise more than 100 muscles and is still solved in fractions of a second on a standard PC for one configuration of an inverse dynamics simulation.

Methods

The min/max muscle recruitment can be stated mathematically as an optimization problem with an objective function, G. We denote the i'th muscle force, $f_i^{(M)}$, and $\mathbf{f}^{(M)}$ is the vector of all muscle forces. We adopt the method introduced by An et. al., 1984, to change the non-smooth min/max problem into a linear problem. An artificial variable, β, that functions as the upper bound on all muscle activities, is introduced. The muscle activity is defined as a relative measure of the muscle force, $f_i^{(M)}/N_i$, where the normalizing factor, N_i, can be taken as the current muscle strength according a Hill-type model. The mathematical definition of the problem then becomes:

Minimize $\quad G(\mathbf{f}^{(M)}) = \beta$

$\mathbf{f}, \beta$ (1)

Subject to
$$\frac{f_i^{(M)}}{N_i} \leq \beta, \quad i \in \{1,\dots,n^{(M)}\} \tag{2}$$

$$f_i^{(M)} \geq 0, \quad i \in \{1,\dots,n^{(M)}\} \tag{3}$$

$$\mathbf{Cf} = \mathbf{d} \tag{4}$$

where Eq. (3) is the additional constraints that muscles cannot push, and Eq. (4) is the dynamics equilibrium equations: $\mathbf{f}$ is the vector of all unknown forces, i.e., muscle as well as reaction force, $\mathbf{d}$ is a vector of all known loads including inertia loads arising due to the motion, and $\mathbf{C}$ is a coefficient matrix. It is observed that the problem is indeed linear and thus can be solved by standard methods; we apply the simplex method for the solution.

As mentioned, the solution is however not always unique. The problems arise from the fact that the min/max formulation only cares about the maximal activity of the muscles. This leaves some indeterminacy because only a subset of the muscles is actually represented in the objective. Indeed, we observe that groups of sub-maximally activated muscles may not be determined uniquely. The problems can be observed even for quite simple models like the one in Fig. 2. It turns out that we can identify three categories of such sub-maximal muscle groups:

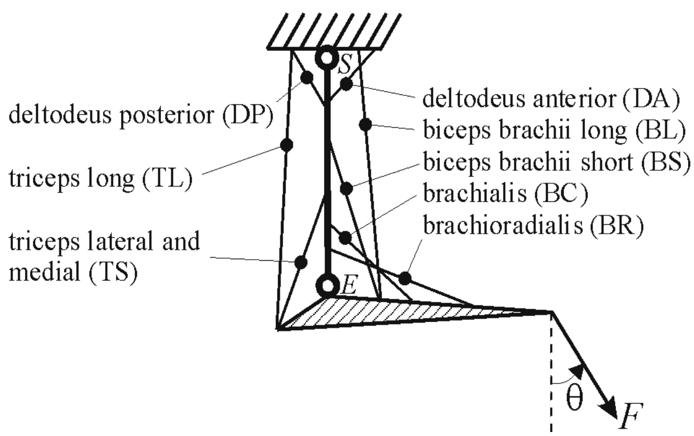

Figure 1: Simple model of the arm.

1. Counter-working muscles
2. Parallel muscles, i.e., muscles with the same function
3. Independent sub-systems, such as one limb in a multi-limb model.

We can attempt to use penalties added to the objective, G, to eliminate the problems. For instance a small linear penalty on any muscle activity can effectively remove category 1 behavior. However, this does not handle the remaining two categories. Non-linear penalties on muscle activity will destroy the linearity of the problem and are therefore not attractive.

Alternatively, we suggest an iterative solution scheme: For each iteration step, we remove the muscles that are determined uniquely from the problem and solve the reduced problem. The iteration is continued until all muscles are determined. Removal of a muscle also implies removing the corresponding loads from the right-hand side of Eq. (4). Please notice that identification of uniquely determined muscles is not a trivial matter; it requires a detailed study of the output from the simplex algorithm.

Results

The three categories of problems with the min/max criterion and their elimination by the iterative solution can be demonstrated by the examples in Figs. 1 and 2. Firstly consider the simple arm in Fig. 2. Table 1 shows relative activities in percent for three cases: Case 1 is the raw min/max solution having counter-working muscles; notice the simultaneous activity of the elbow extensors and flexors. In Case 2, this is eliminated by a linear penalty as suggested above, but the solution is still not satisfactory, because not all elbow flexors are active. However in Case 3, i.e. the iterative solution, all three mono-articular elbow flexors are activated equally. The elbow flexors are so-called parallel muscles, i.e., Category 2.

Muscle name	**Relative activation** [%]		
	Case 1	Case 2	Case 3
deltodeus ant. (DA)	100	100	100
deltodeus post. (DO)	0	0	0
biceps brachii short (BS)	100	0	38
brachialis (BC)	71	27	38
brachioradialis (BR)	100	100	38
triceps lat.+med. (TS)	100	0	0
biceps brachii long (BL)	100	100	100
triceps long (TL)	0	0	0

Table 1: Relative activities for the arm model in Fig. 2 with θ=0. Case 1: Raw min/max according to Eq. (1) to (4). Case 2: Min/max with small linear penalty on muscle activity. Case 3: The iterative solution scheme.

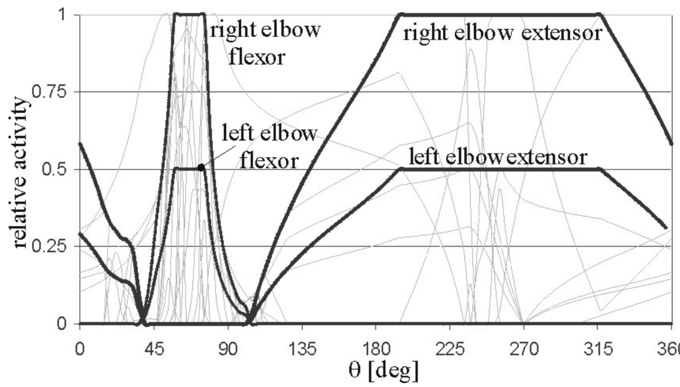

Figure 2: Relative activities over the angle of the load of model in Fig. 1. Double load is applied to the right arm, i.e., $F_1=2F_2$.

Category 3 is found in the model in Fig. 1. One of the arms will be a sub-maximally activated sub-system when a higher force applied to the other. In Fig. 3, we see the relative activities computed by the iterative method for one revolution of the external forces. The model comprises more than 100 muscles, but we have emphasized two, one from each sub-system. These two activation profiles have the same form but their magnitudes are different by a factor two, which is in accordance with the difference of the applied forces. This indicates that the iterative min/max does indeed handle the two sub-systems uniquely, which would not have been the case we the raw min/max solution.

Discussion

Recent work by the authors (Rasmussen et. al., 2001b) has shown that the min/max criterion is equivalent to polynomial criteria of very high power, both qualitatively and mathematically. This is an important point, because the cost of computation of realistic systems with hundreds of muscles will be prohibitive if the objective function is a nonlinear polynomial. The proposed algorithm, on the contrary, handles very large muscle systems on standard PC computers and it is a physiologically reasonable criterion. It is important to notice that the min/max criterion requires such a tailored algorithm, whereas polynomial criteria can be solved by standard optimization methods. We believe that the efficiency of the presented algorithm can make it attractive also in forward dynamics methods and possibly as a way to obtain good starting guesses for iterative solution of non-linear criteria.

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